Magneto-optical phenomena

A magnetic field applied along the wave propagation direction causes changes in the interaction of left and right circularly polarized electromagnetic waves with materials [1,2].

Faraday effect: Different refractive indices for left and right-circularly polarized light can occur at the polarization plane of linearly polarized light.

\[ \Delta n = \frac{N_{0}}{N_{0}^2} \left[ \varphi_{m} - \varphi_{d} \right] \]

Magnetic circular dichroism (MCD): Different absorption of left and right circularly polarized light (transformation of linearly polarized light to elliptically polarized light).

\[ \Delta n = \frac{N_{0}}{N_{0}^2} \left[ \varphi_{m} - \varphi_{d} \right] \]

To the first order in magnetic field, \( \Delta n = \alpha_1 \Delta B \).

For closed shell systems, the expression reduces to:

\[ \Delta n = \alpha_1 \Delta B \]

Problem for periodic systems: The position operator \( \hat{r} \) and, consequently, operators of momenta \( \hat{p} \) and \( \hat{H} \) are ill-defined under periodic conditions.

A new unified approach using a gauge-invariant expression for the position operator should be developed.

Green function in electromagnetic fields

The electromagnetic wave and magnetic field can be considered as perturbations [1,2]:

\[ \hat{H} = \hat{H}_0 + \hat{v} \cdot \hat{E} + \hat{E} \cdot \hat{v} \]

\[ \hat{V} = \hat{V}_0 + \hat{v} \cdot \hat{E} + \hat{E} \cdot \hat{v} \]

\[ \alpha_{\mu} = \alpha_{\mu}^{(0)} \sum \left[ \left( \hat{g}_{\mu \alpha}^{(0)} + \hat{g}_{\mu \alpha}^{(1)} \right) \right] \]

\[ \left( \hat{g}_{\mu \alpha}^{(0)} + \hat{g}_{\mu \alpha}^{(1)} \right) \]

The Green's function of the electromagnetic field [with lattice translational symmetry]

\[ \xi \left( x - x' \right) \]

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In crystal momentum representation:

\[ \sum \left[ \left( \hat{g}_{\mu \alpha}^{(0)} + \hat{g}_{\mu \alpha}^{(1)} \right) \right] \]

Density of states:

\[ \rho_\omega \propto \text{Chem. number} \]

Electromagnetic wave:

\[ \frac{1}{2} \frac{\partial^2}{\partial x^2} \frac{1}{2} \frac{\partial^2}{\partial x^2} \]

Optical response

The polarizability can be extracted from the current response

\[ j_{\alpha}(x) = e^{\alpha_{\mu}^{(0)} + \alpha_{\mu}^{(1)}} \left[ \hat{g}_{\mu \alpha}^{(0)} + \hat{g}_{\mu \alpha}^{(1)} \right] \]

Traditionally, it is assumed that \( \alpha = 0 \) for \( \alpha = 0 \).

However, if \( \alpha = 0 \), it is more convenient to derive the response using a different formulation.

According to the properties of Fourier transform, for any function \( F \)

\[ \left[ \hat{F}_{\mu \alpha}^{(0)} + \hat{F}_{\mu \alpha}^{(1)} \right] \]

Let us introduce the position "superoperator" that acts on whole matrix elements

\[ \left( \hat{g}_{\mu \alpha}^{(0)} + \hat{g}_{\mu \alpha}^{(1)} \right) \]

Beryllium \( \alpha \) Anomalous Hall effect (AHE)

Optical response for biaxial systems

Magneto-optical response

Electromagnetic wave (uniform magnetic field)

\[ \alpha_{\mu} \alpha_{\nu} \]