Predicting the critical temperature of Superconductors with no experimental input has proven to be really difficult.

More than 50 year after the BCS theory of superconductivity many important classes of superconductors like cuprates and the iron family are not completely understood at a fundamental level.

Eliashberg theory of superconductivity, that is essentially a $GW_0$ method in the off diagonal Nambu channel, has been so far the most successful and popular approach for phononic superconductivity. Although the coulomb part of the $W$ is modeled and reduced to a fitting parameter in the “so called” $\mu^*$ approach.

Density Functional Theory for Superconductors (SCDFT), on the other hand, accounts (within $GW_0$) for both Coulomb and Phonon interactions on the same footing, at the same time skipping the complexity of tedious Matsubara sums.

However, missing vertex corrections limit its applicability to phononic superconductors.

In this talk we review these approaches, most recent functionals, computational schemes and present results on new and classic superconductors. We discuss the role of spin fluctuations and the structure of pairing interactions in Iron based superconductors.