7. General principles of TDDFT (2 points)

a) Can more than one potential produce the same time-dependent density?

b) If a system is subjected to a driving potential that, after a time, returns its density to its initial value, will the potential return to its initial value?

c) Answer question b) for an adiabatic approximation.

8. Adiabatic LDA exchange (2 points)

The LDA exchange functional in 3 dimensions is given as

$$E_x^{\mathrm{LDA}}[\rho] = -\frac{3}{4}\left(\frac{3}{\pi}\right)^{1/3}\int d^3r\,\rho^{4/3}(\mathbf{r}).$$

a) Calculate the adiabatic LDA exchange potential $v_x^{\mathrm{ALDA}}(\mathbf{r},t)$.

b) Calculate the corresponding exchange kernel $f_x^{\mathrm{ALDA}}(\mathbf{r},\mathbf{r}',t,t')$.

c) Which property of f_x^{ALDA} shows that the approximation is adiabatic? Which property shows that it is a local approximation?

9. Dipole moment (2 points)

The dipole moment of a single particle in state $\phi(\mathbf{r},t)$ is given as

$$\mathbf{D}(t) = \int d^3r\,\phi^*(\mathbf{r},t)\mathbf{r}\phi(\mathbf{r},t).$$

a) Give the corresponding equation for the calculation of the dipole moment of N particles in state $\psi(\mathbf{r}_1...\mathbf{r}_N,t)$.

b) Show that one can rewrite the result of a) as

$$\mathbf{D}(t) = \int d^3r\,\mathbf{r}\rho(\mathbf{r},t).$$